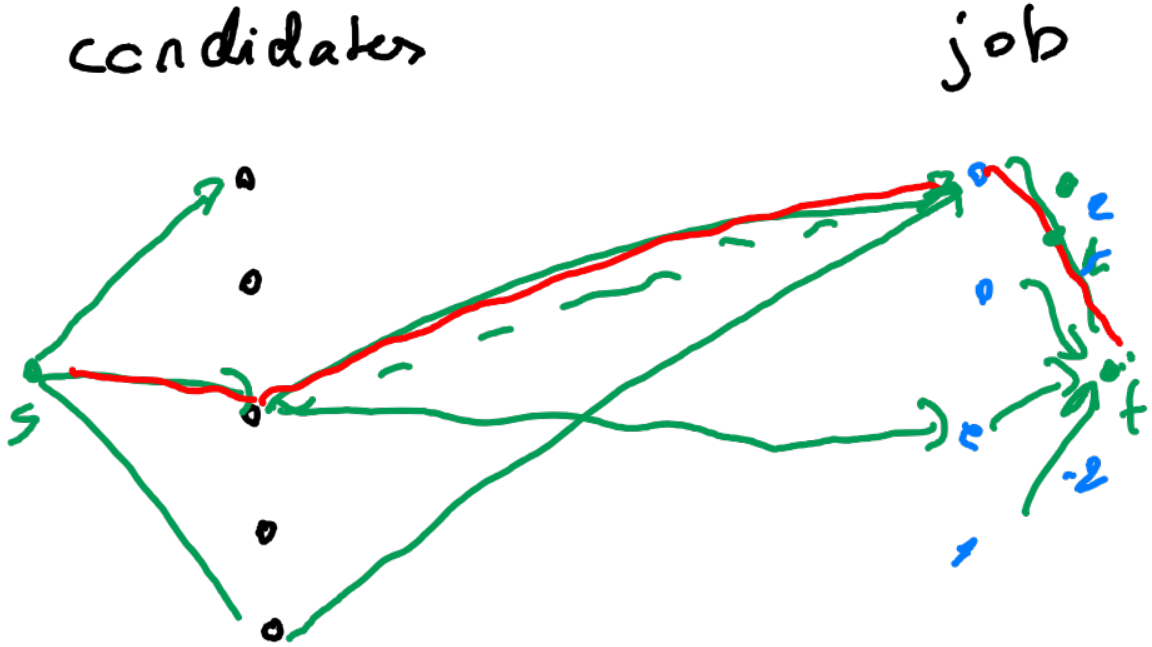
Announcements

- Regrade requests are due a week after we open the grades.
- OH Eva Tardos:
  Tuesdays: 1:30-2:30
  Fridays 10:15-11:15
- Study abroad program ~~problem~~ opportunity focusing in CS

Problem 2: assigning candidates to companies



① assign as many candidates as you can at ≥1 candidate

② get as many jobs as you can at ≥1 candidate ✓

start ② matching

next ① update capacity to continue

notice edges (v,t) never used backwards

⟹ keep first objective untouched

Homework Policy (reminder)

- You are encouraged to only reference course materials in developing your solutions. If you choose to consult written sources beyond the course material and textbook (including Generative AI), you must **cite each source clearly in your submission**, if using Generative AI need to include the transcript of your interaction. Do not solicit help from people outside of the class (beyond classmates and course staff).

do not use stuff you don't understand

Hardness of problems and Reductions

Problems X & Y $\quad X \leq_p Y$ - : X is no harder than Y for polynomial time

typical proof: algorithm for Y can solve X $\Longleftrightarrow$ reduction

hw1: road scheduling on island $\leq_p$ MST ✓

- max matching $\leq_p$ max flow (in integers)

project scheduling $\leq_p$ max flow ✓

hw2. event scheduling $\leq$ min cost path ✓

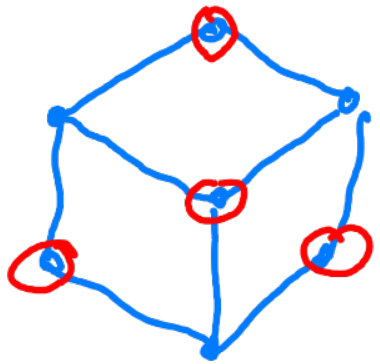
Hardness of problems: Independent Set

Input undirected graph $G = (V, E)$

find $I \subseteq V$ such that $v, w \in I$ not $(v, w) \notin E$

$|I|$ maximum

Example

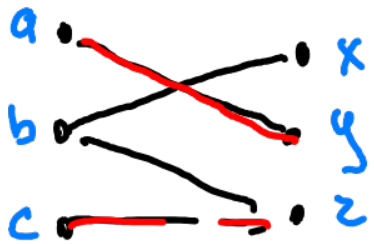


indep set of size 4

Claim max matching $\leq_p$ Indep Set

Proof:

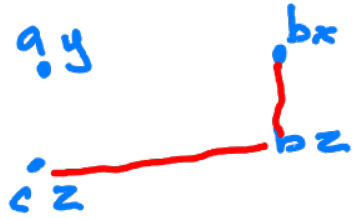
Input matching



$G = (V, E)$

$\Rightarrow$

G' nodes = E



edges: if they share node

Claim: indep set in G' in one to one correspondence with matchings in G

Join by Web **PollEv.com/evatardos772**

- What is the maximum size independent set in the graph below?

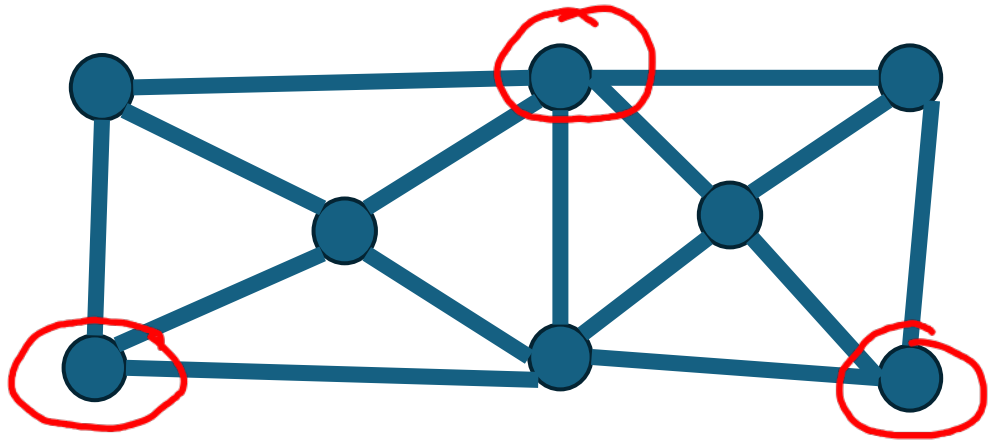
indep set
$I \subseteq V$, no two connected by an edge

A: 1

B: 2

C: 3

D: 4



E: I don't understand what independent set is

Hardness of problems: Independent Set

Interval scheduling $\leq_p$ Independent set

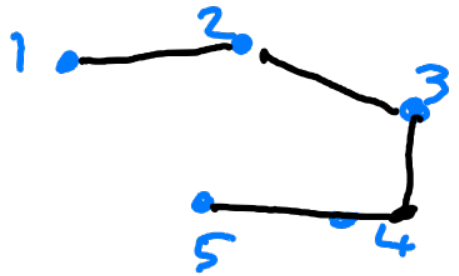
input 

select as many as possible disjoint intervals

Solved via greedy

Create input Indep set

V = intervals, edges (i,j) if intersect



Claim: disjoint set of intervals are in one to one correspondence to independent sets in G

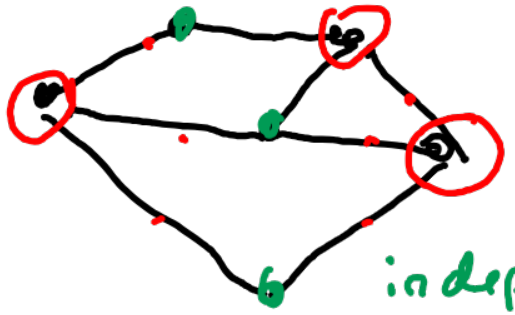
Coming Wednesday

Problem is NP-complete

Hardness of problems: Independent Set

Proving a new problem hard: vertex cover

Input $G = (V, E)$ undirected



e.g vertex cover with 3 nodes

indep set

$S \subseteq V$ vertex cover is S contains at least one end of every edge

find vertex cover of smallest size

Claim Indep Set $\leq_p$ Vertex Cover

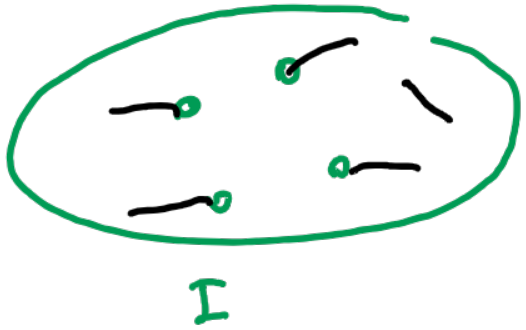
If we had an algorithm for Vertex Cover we could use that to solve Independ Set.

Proof

input Indep set $G = (V, E)$ ⟶ input for vertex cover use same graph $G = (V, E)$

Proving a new problem hard: vertex cover

Note: Indepdent set $I \iff V \setminus I$ is vertex cover



$I \Rightarrow V \setminus I$ vertex cover

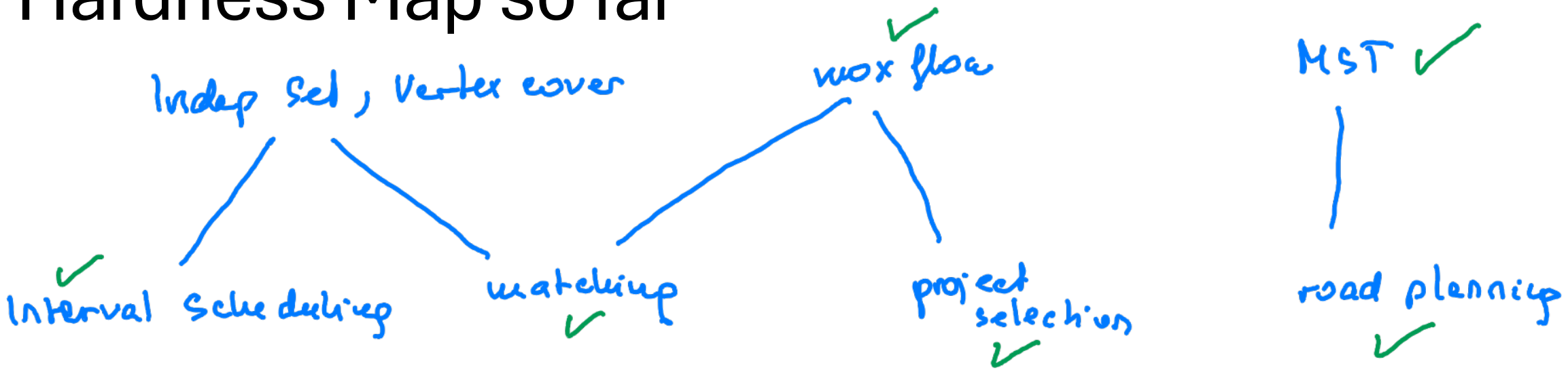
$S = V \setminus I$ vertex cover $\Rightarrow I$ indep

Using this fact min vertex covers size k if & only if max indep set size $|V|-k$

In fact Vertex cover $\leq_p$ Indep Set

by the same proof.

Hardness Map so far



polynomial time solvable

P = [yes/no decision problems solvable in polynomial time]

✓ above ⇒ decision version in P

Indep Set decision version

G = (V, E) & integer k

Is there an indep set of size k

Add target size / cost to all optimization problems